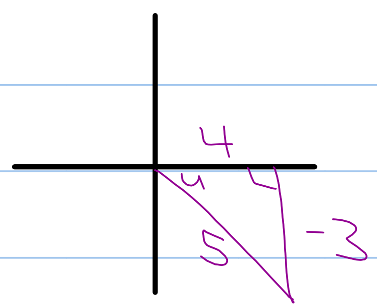
Section 5.1 Using Fundamental Identities

Example 1. If csc $u = -5/3$ and cos $u > 0$, find the values of the other five trigonometric functions.



$\sin u = \frac{-3}{5}$

$\cos u = \frac{4}{5}$ $\qquad \sec u = \frac{5}{4}$

$\tan u = \frac{-3}{4}$ $\qquad \cot u = \frac{-4}{3}$

Example 2. Simplify the following.

a) $\csc^2 x \cot x - \cot x$

$= \cot x(\csc^2 x - 1) = \cot x(\cot^2 x) = \boxed{\cot^3 x}$

b) $\tan x \sin x + \cos x$

$= \frac{\sin x}{\cos x} \cdot \sin x + \cos x$

$= \frac{\sin^2 x}{\cos x} + \frac{\cos x}{1} \cdot \frac{\cos x}{\cos x} = \frac{\sin^2 x + \cos^2 x}{\cos x} = \frac{1}{\cos x} = \boxed{\sec x}$

c) $\dfrac{\sec t}{\tan t} - \dfrac{\tan t}{1+\sec t}$

$= \frac{(\sec t)(1+\sec t)}{(\tan t)(1+\sec t)} - \frac{(\tan t)(\tan t)}{(\tan t)(1+\sec t)}$

$= \frac{\sec t + \sec^2 t - \tan^2 t}{\tan t(1+\sec t)} = \frac{\cancel{\sec t + 1}}{\tan t(\cancel{1+\sec t})} = \frac{1}{\tan t}$

$= \cot t$

Example 3. Factor the following trigonometric expressions.

a) $\cos^2 x - 1$ $= (\cos x)^2 - (1)^2$

$= (\cos x + 1)(\cos x - 1)$

b) $\sin^2 u - 3\sin u - 10$

$= (\sin u - 5)(\sin u + 2)$

c) $\sec^2 t - \tan t - 3$

$= 1 + \tan^2 t - \tan t - 3$

$= \tan^2 t - \tan t - 2$

$= (\tan t - 2)(\tan t + 1)$

Example 4. Rewrite $\dfrac{1}{1-\sec x}$ so that it is not a fraction.

Multiply by Conjugate of Denominator

$$\frac{1}{(1-\sec x)}\frac{(1+\sec x)}{(1+\sec x)} = \frac{1+\sec x}{1-\sec^2 x} = \frac{1+\sec x}{1-(1+\tan^2 x)}$$

$$\frac{1}{\tan x} = \cot x \qquad = \frac{1+\sec x}{-\tan^2 x} = \frac{-1}{\tan^2 x} - \frac{\sec x}{\tan^2 x}$$

$$= -\cot^2 x - \frac{1}{\tan^2 x}\cdot \sec x$$

$$= \boxed{-\cot^2 x - \cot^2 x \sec x}$$

Example 5. Use the substitution $x = 3\sin u$, $0 < u < \pi/2$, to express $\sqrt{9 - x^2}$ as a function of u and simplify.

$$= \sqrt{9 - (3\sin u)^2} = \sqrt{9 - 9\sin^2 u}$$

$$= \sqrt{9(1 - \sin^2 u)} = \sqrt{9\cos^2 u} = \sqrt{9} \cdot \sqrt{\cos^2 u}$$

$$= \boxed{3\cos u}$$