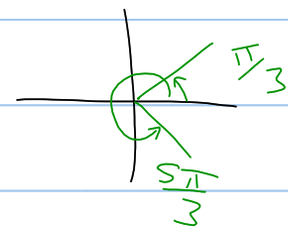
Section 5.3 Solving Trigonometric Equations

Example 1. Solve $1-2\cos x=0$ where $0 \le x < 2\pi$

$-2\cos x = -1$

$\rightarrow \cos x = \frac{1}{2}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$



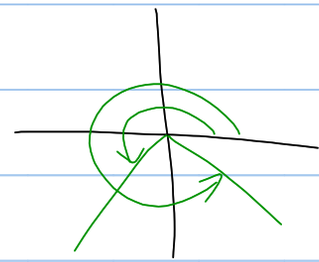
Example 2. Solve $\sin x + 1 = -\sin x$ where $0^o \le x < 360^o$

$-\sin x \quad -\sin x$

$1 = -2\sin x$

$\sin x = -\frac{1}{2}$

$x = 210^\circ, 330^\circ$

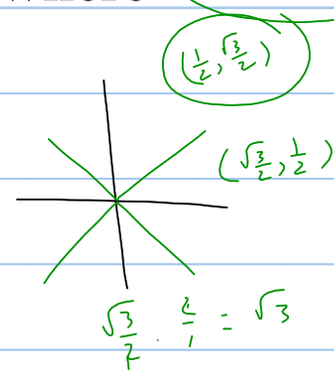


Example 3. Solve $\tan^2 x - 3 = 0$ where $0 \le x \le 2\pi$

$\tan^2 x = 3$

$\tan x = \pm\sqrt{3}$

$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$



Example 4. Solve $\sec x \sin x = \sin x$ where $0^o \le x < 360^o$

$-\sin x \quad -\sin x$

$\sec x \sin x - \sin x = 0$

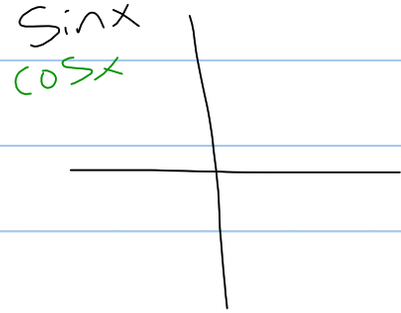
$\sin x(\sec x - 1) = 0$

$\sin x = 0 \qquad \sec x - 1 = 0$

$x = 0°, 180° \qquad \sec x = 1$

$x = 0° \qquad \frac{1}{\cos x} = 1$

$\cos x = 1$



Example 5. Solve the following on the interval $[0, 2\pi)$.

a) $2\cos^2 x + \cos x - 1 = 0$

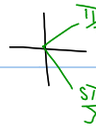
$2(\cos x)^2 + \cos x - 1 = 0$

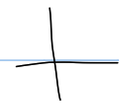
$(2\cos x - 1)(\cos x + 1) = 0$

$2\cos x - 1 = 0 \quad \cos x + 1 = 0$

$\cos x = \frac{1}{2} \quad \cos x = -1$

$x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$



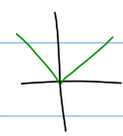


b) $2\cos^2 x + 3\sin x - 3 = 0$

$2(1 - \sin^2 x) + 3\sin x - 3 = 0$

$2 - 2\sin^2 x + 3\sin x - 3 = 0$

$-1(-2\sin^2 x + 3\sin x - 1) = (0) - 1$



$2\sin^2 x - 3\sin x + 1 = 0$

$(2\sin x - 1)(\sin x - 1) = 0$

$2\sin x - 1 = 0 \quad \sin x - 1 = 0$

$\sin x = \frac{1}{2} \quad \sin x = 1$

$x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$

c) $(\sec x + 1)^2 = (\tan x)^2$

$\sec^2 x + 2\sec x + 1 = \tan^2 x$

$\sec^2 x + 2\sec x + 1 = \sec^2 x - 1$

$2\sec x + 2 = 0$

$\sec x = -1$

$\frac{1}{\cos x} = -1$

$\cos x = -1$

$x = \pi$

Example 6. Solve the following on the interval $[0, 2\pi)$.

a) $2\sin(2t)+1=0$

When the variable inside a cosine, sine secant, or Cosecant is multiplied by a # we need to add $2\pi n$ to the angle.

$\sin(2t) = -\frac{1}{2}$

$2t = \frac{7\pi}{6} + 2\pi n$ $\quad$ $2t = \frac{11\pi}{6} + 2\pi n$

Solve for t

$t = \frac{7\pi}{12} + \frac{12\pi}{12}n$ $\quad$ $t = \frac{11\pi}{12} + \frac{12\pi}{12}n$

n=0: $\frac{7\pi}{12}, \frac{11\pi}{12}$

n=1: $\frac{19\pi}{12}, \frac{23\pi}{12}$

b) $\cot\left(\frac{x}{2}\right)+1=0$

When the variable inside a tangent or cotangent is multiplied by a # we need to add πn to the angle.

$\cot(\frac{x}{2}) = -1$

$\frac{x}{2} = \frac{3\pi}{4} + \pi n$ $\quad$ $\frac{x}{2} = \frac{7\pi}{4} + \pi n$

$x = \frac{3\pi}{2} + 2\pi n$ $\quad$ $x = \frac{7\pi}{2} + 2\pi n$

n=0: $\frac{3\pi}{2}$, ~~$\frac{7\pi}{2}$~~