Chapter 6
Differential Equations and Mathematical Modeling

Section 6.1 Slope Fields and Euler's Method (pp. 321–330)

Exploration 1 Seeing the Slopes

1. Since $\frac{dy}{dx}=0$ represents a line with a slope of 0, we should expect to see intervals with no change in y. We see this at odd multiples of $\pi/2$.

[redacted]

3. The graph of $\frac{dy}{dx}$ will look the same at all values of y.

[redacted]

5. When $x=\pi$, $\frac{dy}{dx}=\cos x=-1$. This can be seen in the graph at $x=\pi$. At this point, the change in y is negative of the change in x.

[redacted]

Quick Review 6.1

1. Yes. $\frac{d}{dx}e^x=e^x$

[redacted]

3. No. $\frac{d}{dx}(x^2e^x)=2xe^x+x^2e^x$

[redacted]

5. No. $\frac{d}{dx}(e^{x^2}+5)=2xe^{x^2}$

[redacted]

7. Yes. $\frac{d}{dx}\sec x=\sec x\tan x$

[redacted]

9. $y=3x^2+4x+C$
$2=3(1)^2+4(1)+C$
$C=-5$

[redacted]

11. $y=e^{2x}+\sec x+C$
$5=e^{2(0)}+\sec(0)+C$
$C=3$

[redacted]

Section 6.1 Exercises

1. $\int dy=\int(5x^4-\sec^2 x)\,dx$
$y=x^5-\tan x+C$

[redacted]

3. $\int dy=\int(\sin x-e^{-x}+8x^3)\,dx$
$y=-\cos x+e^{-x}+2x^4+C$

[redacted]

5. $\int dy=\int\left(5^x\ln 5+\frac{1}{x^2+1}\right)dx=5^x+\tan^{-1}x+C$

[redacted]

7. $\int dy=\int(3t\cos(t^3))\,dt=\sin(t^3)+C$

[redacted]

9. $\int dy=\int(\sec^2(x^5)(5x^4))\,dx$
$=\tan x^5+C$

[redacted]

11. $\int dy=\int 3\sin x\;dx=-3\cos x+C$
$2=-3\;\cos(0)+C,\quad C=5$
$y=-3\;\cos x+5$

[redacted]

13. $\int du=\int(7x^6-3x^2+5)\,dx=x^7-x^3+5x+C$
$1=1^7-1^3+5+C,\quad C=-4$
$u=x^7-x^3+5x-4$

[redacted]

15. $\int dy = \int\left(-\frac{1}{x^2}-\frac{3}{x^4}+12\right)\,dx = x^{-1}+x^{-3}+12x+c$

$3 = 1^{-1}+1^{-3}+12(1)+C, \quad C=-11$

$y = x^{-1}+x^{-3}+12x-11 \ \ (x>0)$

[redacted]

17. $\int dy = \int\left(\frac{1}{1+t^2}+2^t\ln 2\right)dt = \tan^{-1}t+2^t+C$

$3 = \tan^{-1}(0)+2^0+C, \quad C=2$

$y = \tan^{-1}t+2^t+C$

[redacted]

19. $\int dv = \int\left(4\sec t\ \tan t+e^t+6t\right)\,dt = 4\sec t+e^t+3t^2+C$

$5 = 4\ \sec(0)+e^0+3(0)^2+C, \quad C=0$

$V = 4\ \sec t+e^t+3t^2 \quad \left(-\frac{\pi}{2}<t<\frac{\pi}{2}\right)$

[redacted]

21. $\frac{dy}{dx}=\frac{d}{dx}\int_a^x f(t)\,dt = \int_1^x \sin(t^2)\,dt$

$y=\int_1^x \sin(t^2)\,dt+5$

[redacted]

23. $F^1(x)=\frac{d}{dx}\int_a^x f(t)\,dt = \int_2^x e^{\cos t}dt$

$F(x)=\int_2^x e^{\cos t}dt+9$

[redacted]

25. Graph (b).

$(\sin 0)^2=0$

$(\sin 1)^2>0$

$(\sin(-1))^2>0$

[redacted]

27. Graph (a).

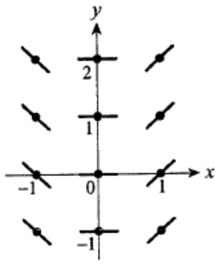
$(\cos 0)^2>0$

$(\cos 1)^2>0$

$(\cos(-1)^2>0$

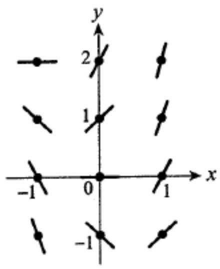
[redacted]

29.



[redacted]

31.



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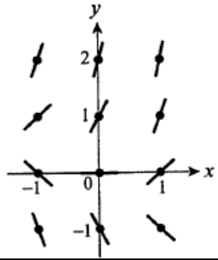
33.



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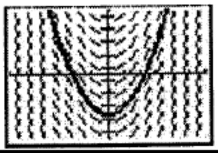
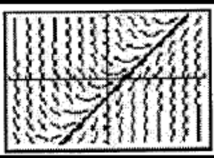
35.



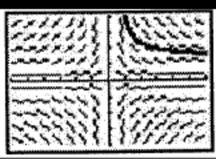
[redacted]

37.



[redacted]

39.



[redacted]

41.

(x, y)	$\frac{dy}{dx} = x - 1$	Δx	$\Delta y = \frac{dy}{dx}\Delta x$	$(x + \Delta x, y + \Delta y)$
(1, 2)	0.0	0.1	0	(1.1, 2)
(1.1, 2)	0.1	0.1	0.01	(1.2, 2.01)
(1.2, 2.01)	0.2	0.1	0.02	(1.3, 2.03)

$y = 2.03$

[redacted]

43.

(x, y)	$\frac{dy}{dx} = 2x - y$	Δx	$\Delta y = \frac{dy}{dx}\Delta x$	$(x + \Delta x, y + \Delta y)$
(1, 2)	1.0	0.1	0.1	(1.1, 2.1)
(1.1, 2.1)	1.0	0.1	0.1	(1.2, 2.2)
(1.2, 2.2)	1.0	0.1	0.1	(1.3, 2.3)

$y = 2.3$

[redacted]

45.

(x, y)	$\frac{dy}{dx} = 2 - x$	Δx	$\Delta y = \frac{dy}{dx}\Delta x$	$(x + \Delta x, y + \Delta y)$
(2, 1)	0.0	−0.1	0.0	(1.9, 1)
(1.9, 1)	0.1	−0.1	−0.01	(1.8, 0.99)
(1.8, 0.99)	0.2	−0.1	−0.02	(1.7, 0.97)

[redacted]

47.

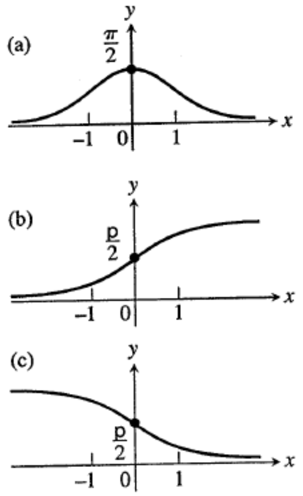
(x, y)	$\frac{dy}{dx} = x - y$	Δx	$\Delta y = \frac{dy}{dx}\Delta x$	$(x + \Delta x, y + \Delta y)$
(2, 2)	−0.0	−0.1	0	(1.9, 2.0)
(1.9, 2)	−0.1	−0.1	0.01	(1.8, 2.01)
(1.8, 2.01)	−0.21	−0.1	0.021	(1.7, 2.031)

$y = 2.031$

[redacted]

49. **(a)** Graph (b)

(b) The slope is always positive, so (a) and (c) can be ruled out.



[redacted]

51. There are positive slopes in the second quadrent of the slope field. The graph of $y = x^2$ has negative slopes in the second quadrent.

[redacted]

53.

(x, y)	$\frac{dy}{dx} = 2x + 1$	Δx	$\Delta y = \frac{dy}{dx}\Delta x$	$(x + \Delta x, y + \Delta y)$
(1, 3)	3.0	0.1	0.3	(1.1, 3.3)
(1.1, 3.3)	3.2	0.1	0.32	(1.2, 3.62)
(1.2, 3.62)	3.4	0.1	0.34	(1.3, 3.96)
(1.3, 3.96)	3.6	0.1	0.36	(1.4, 4.32)

$y = 4.32$

Euler's Method gives an estimate $f(1.4) \approx 4.32$.
The solution to the initial value problem is $f(x) = x^2 + x + 1$, from which we get $f(1.4) = 4.36$. The percentage error is thus $\frac{4.36 - 4.32}{4.36} = 0.9\%$.

[redacted]

55. At every (x, y), $(-e^{(x-y)/2})(-e^{(y-x)/2}) = -e^0 = -1$, so the slopes are negative reciprocals. The slope lines are therefore perpendicular.

[redacted]

57. The perpendicular slope field would be produced by $\frac{dy}{dx} = -\sin x$, so $y = \cos x + C$ for any constant C.

[redacted]

59. True. They are all lines of the form $y = 5x + C$.

[redacted]

61. C. $m = 42 - 42 = 0$

[redacted]

63. B. $y(0) = e^{0^2} = 1$

$$\frac{dy}{dx} = 2xe^{x^2} = 2xy.$$

[redacted]

65. (a) $\dfrac{dy}{dx}=x-\dfrac{1}{x^2}$

$$\int \frac{dy}{dx}\,dx=\int (x-x^{-2})\,dx$$

$$y=\frac{x^2}{2}+x^{-1}+C=\frac{x^2}{2}+\frac{1}{x}+C$$

Initial condition: $y(1)=2$

$$2=\frac{1^2}{2}+\frac{1}{1}+C$$

$$2=\frac{3}{2}+C$$

$$\frac{1}{2}=C$$

Solution: $y=\dfrac{x^2}{2}+\dfrac{1}{x}+\dfrac{1}{2},\ x>0$

(b) Again, $y=\dfrac{x^2}{2}+\dfrac{1}{x}+C.$

Initial condition: $y(-1)=1$

$$1=\frac{(-1)^2}{2}+\frac{1}{(-1)}+C$$

$$1=\frac{-1}{2}+C$$

$$\frac{3}{2}=C$$

Solution: $y=\dfrac{x^2}{2}+\dfrac{1}{x}+\dfrac{3}{2},\ x<0$

(c) For $x<0$, $\dfrac{dy}{dx}=\dfrac{d}{dx}\left(\dfrac{1}{x}+\dfrac{x^2}{2}+C_1\right)$

$$=-\frac{1}{x^2}+x$$

$$=x-\frac{1}{x^2}.$$

For $x>0$, $\dfrac{dy}{dx}=\dfrac{d}{dx}\left(\dfrac{1}{x}+\dfrac{x^2}{2}+C_2\right)$

$$=-\frac{1}{x^2}+x$$

$$=x-\frac{1}{x^2}.$$

And for $x=0$, $\dfrac{dy}{dx}$ is undefined.

(d) Let C_1 be the value from part (b), and let C_2 be the value from part (a). Thus, $C_1=\dfrac{3}{2}$ and $C_2=\dfrac{1}{2}$.

(e) $y(2)=-1$ $\qquad$ $y(-2)=2$

$$-1=\frac{1}{2}+\frac{2^2}{2}+C_2 \qquad 2=\frac{1}{(-2)}+\frac{(-2)^2}{2}+C_1$$

$$-1=\frac{5}{2}+C_2 \qquad 2=\frac{3}{2}+C_1$$

$$-\frac{7}{2}=C_2 \qquad \frac{1}{2}=C_1$$

Thus, $C_1=\dfrac{1}{2}$ and $C_2=-\dfrac{7}{2}$.

[redacted]

67. (a) $y'=\int 12x+4\,dx$

$$y'=6x^2+4x+C_1$$

$$y=\int 6x^2+4x+C_1\,dx$$

$$y=2x^3+2x^2+C_1x+C_2$$

(b) $y'=\int e^x+\sin x\,dx$

$$y'=e^x-\cos x+C_1$$

$$y=\int e^x-\cos x+C_1\,dx$$

$$y=e^x-\sin x+C_1x+C_2$$

(c) $y'=\int x^3+x^{-3}\,dx$

$$y'=\frac{x^4}{4}-\frac{1}{2x^2}+C_1$$

$$y=\int \frac{x^4}{4}-\frac{1}{2x^2}+C_1\,dx$$

$$y=\frac{x^5}{20}+\frac{1}{2x}+C_1x+C_2$$

[redacted]

69. (a) $y' = x$

$$y = \int x\,dx = \frac{x^2}{2} + C$$

(b) $y' = -x$

$$y = \int -x\,dx = -\frac{x^2}{2} + C$$

(c) $y' = y$

$$\frac{d}{dx}(Ce^x) = Ce^x$$
$$y = Ce^x$$

(d) $y' = -y$

$$\frac{d}{dx}(Ce^{-x}) = -Ce^{-x}$$
$$y = Ce^{-x}$$

(e) $y' = xy$

$$\frac{d}{dx}\left(Ce^{x^2/2}\right) = Cxe^{x^2/2}$$
$$y = Ce^{x^2/2}$$

[redacted]

Section 6.2 Antidifferentiation by Substitution (pp. 331–340)

Exploration 1 Are $\int f(u)\,du$ and $\int f(u)\,dx$ the Same Thing?

1. $\int f(u)\,du = \int u^3 du$

$$= \frac{u^4}{4} + C$$

[redacted]

3. $f(u) = u^3 = (x^2)^3 = x^6$

$$\int x^6 dx = \frac{x^7}{7}$$

[redacted]

Quick Review 6.2

1. $\int_0^2 x^4 dx = \frac{1}{5}x^5\Big]_0^2 = \frac{1}{5}(2)^5 - \frac{1}{5}(0)^5 = \frac{32}{5}$

[redacted]

3. $\frac{dy}{dx} = 3^x$

[redacted]

5. $\frac{dy}{dx} = 4(x^3 - 2x^2 + 3)^3(3x^2 - 4x)$

[redacted]

7. $\frac{dy}{dx} = \frac{1}{\cos x} \bullet -\sin x = -\tan x$

[redacted]

9. $\frac{dy}{dx} = \frac{1}{\sec x + \tan x} \bullet (\sec x \tan x + \sec^2 x)$

$= \frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x}$

$= \frac{\sec x(\tan x + \sec x)}{\sec x + \tan x}$

$= \sec x$

[redacted]

Section 6.2 Exercises

1. $\int (\cos x - 3x^2)\, dx = \sin x - x^3 + C$

[redacted]

3. $\int \left(t^2 - \frac{1}{t^2}\right) dt = \frac{t^3}{3} + t^{-1} + C$

[redacted]

5. $\int (3x^4 - 2x^{-3} + \sec^2 x)\, dx = \frac{3}{5}x^5 + x^{-2} + \tan x + C$

[redacted]

7. $(-\cot u + C)' = -(-\csc^2 u) = \csc^2 u$

[redacted]

9. $\left(\frac{1}{2}e^{2x} + C\right)' = \frac{1}{2}e^{2x}(2) = e^{2x}$

[redacted]

11. $(\tan^{-1} u + C)' = \frac{1}{1+u^2}$

[redacted]

13. $\int f(u)\, du = \int \sqrt{u}\, du = \frac{2}{3}u^{3/2} + C = \frac{2}{3}x^{3/2} + C$

$\int f(u)\, dx = \int \sqrt{u}\, dx = \int \sqrt{x^2} dx = \int x\, dx = \frac{1}{2}x^2 + C$

[redacted]

15. $\int f(u)\, du = \int e^u\, du = e^u + C = e^{7x} + C$

$\int f(u)\, du = \int e^u dx = \int e^{7x} dx = \frac{1}{7}e^{7x} + C$

[redacted]

17. $u = 3x$

$du = 3\, dx$

$\frac{1}{3} du = dx$

$\int \sin 3x\, dx = \frac{1}{3}\int \sin u\, du$

$= -\frac{1}{3}\cos u + C$

$= -\frac{1}{3}\cos 3x + C$

Check: $\frac{d}{dx}\left(-\frac{1}{3}\cos 3x + C\right) = \frac{1}{3}(-\sin 3x)(3) = \sin 3x$

[redacted]

19. $u = 2x$

$du = 2\ dx$

$\frac{1}{2}du = dx$

$$\int \sec 2x \tan 2x\ dx = \frac{1}{2}\int \sec u \tan u\ du$$
$$= \frac{1}{2}\sec u + C$$
$$= \frac{1}{2}\sec 2x + C$$

Check: $\frac{d}{dx}\left(\frac{1}{2}\sec 2x + C\right) = \frac{1}{2}\sec 2x \tan 2x \cdot 2 = \sec 2x \tan 2x$

[redacted]

21. $u = \frac{x}{3}$

$du = \frac{1}{3}dx$

$3\ du = dx$

$$\int \frac{dx}{x^2+9} = \int \frac{3du}{9u^2+9}$$
$$= \frac{3}{9}\int \frac{du}{u^2+1}$$
$$= \frac{1}{3}\int \frac{du}{u^2+1}$$
$$= \frac{1}{3}\tan^{-1}u + C$$
$$= \frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + C$$

Check: $\frac{d}{dx}\left(\frac{1}{3}\tan^{-1}\frac{x}{3} + C\right) = \frac{1}{3}\frac{1}{1+\left(\frac{x}{3}\right)^2} \cdot \frac{1}{3} = \frac{1}{9+x^2}$

[redacted]

23. $u = 1 - \cos\frac{t}{2}$

$du = \frac{1}{2}\sin\frac{t}{2}dt$

$2\ du = \sin\frac{t}{2}dt$

$$\int \left(1-\cos\frac{t}{2}\right)^2 \sin\frac{t}{2}dt = 2\int u^2 du$$
$$= \frac{2}{3}u^3 + C$$
$$= \frac{2}{3}\left(1-\cos\frac{t}{2}\right)^3 + C$$

Check: $\frac{d}{dx}\left[\frac{2}{3}\left(1-\cos\frac{t}{2}\right)^3 + C\right]$

$$= 2\left(1-\cos\frac{t}{2}\right)^2\left(\sin\frac{t}{2}\right)\left(\frac{1}{2}\right)$$
$$= \left(1-\cos\frac{t}{2}\right)^2 \sin\frac{t}{2}$$

[redacted]

[redacted]

$du = -dx$

$\int \frac{dx}{(1-x^2)} = -\int \frac{du}{u^2}$

$= u^{-1} + C$

$= \frac{1}{1-x} + C$

[redacted]

27. Let $u = \tan x$

$du = \sec^2 x\ dx$

$\int \sqrt{\tan x} \sec^2 x\ dx = \int u^{1/2} du$

$= \frac{2}{3} u^{3/2} + C$

$= \frac{2}{3} (\tan x)^{3/2} + C$

[redacted]

29. $\int \tan(4x+2)\ dx$

$u = 4x+2$

$du = 4\ dx$

$\frac{1}{4} du = dx$

$\frac{1}{4} \int \tan u\ du$

$= -\frac{1}{4} \ln|\cos(4x+2)| + C$ or

$\frac{1}{4} \ln|\sec(4x+2)| + C$

[redacted]

31. Let $u = 3z+4$

$du = 3\ dz$

$\frac{1}{3} du = dz$

$\int \cos(3z+4) dz = \frac{1}{3} \int \cos u\ du$

$= \frac{1}{3} \sin u + C$

$= \frac{1}{3} \sin(3z+4) + C$

[redacted]

33. Let $u = \ln x$

$du = \frac{1}{x} dx$

$\int \frac{\ln^6 x}{x} dx = \int u^6 du$

$= \frac{1}{7} u^7 + C$

$= \frac{1}{7} (\ln^7 x) + C$

[redacted]

35. Let $u = s^{4/3} - 8$

$du = \frac{4}{3} s^{1/3} ds$

$\frac{3}{4} du = s^{1/3} ds$

35. Continued

$$\int s^{1/3}\cos(s^{4/3}-8)\,ds = \frac{3}{4}\int \cos u\,du$$
$$= \frac{3}{4}\sin u + C$$
$$= \frac{3}{4}\sin(s^{4/3}-8) + C$$

[redacted]

37. Let $u = \cos(2t+1)$

$du = -\sin(2t+1)(2)dt$

$-\frac{1}{2}du = \sin(2t+1)dt$

$$\int \frac{\sin(2t+1)}{\cos^2(2t+1)}dt = -\frac{1}{2}\int u^{-2}du$$
$$= \frac{1}{2}u^{-1} + C$$
$$= \frac{1}{2\cos(2t+1)} + C$$
$$= \frac{1}{2}\sec(2t+1) + C$$

[redacted]

39. $\int \frac{dx}{x \ln x}$

$u = \ln x$

$du = \frac{dx}{x}$

$x\,du = dx$

$$\int \frac{du}{u} = \ln u = \ln(\ln x) + C$$

[redacted]

41. $\int \frac{x\,dx}{x^2+1}$

$u = x^2 + 1$

$du = 2x\,dx$

$\frac{du}{2x} = dx$

$$\frac{1}{2}\int \frac{du}{x^2+1} = \frac{1}{2}\ln u + C$$
$$= \frac{1}{2}\ln(x^2+1) + C$$

[redacted]

43. $\int \frac{dx}{\cot 3x} = \int \frac{\sin 3x}{\cos 3x}dx$

Let $u = \cos 3x$

$du = -3\sin 3x\,dx$

$-\frac{1}{3}du = \sin 3x\,dx$

$$\int \frac{dx}{\cot 3x} = -\frac{1}{3}\int \frac{1}{u}du$$
$$= -\frac{1}{3}\ln|u| + C$$
$$= -\frac{1}{3}\ln|\cos 3x| + C$$

(An equivalent expression is $\frac{1}{3}\ln|\sec 3x| + C$.)

[redacted]

45. $\int \sec x\,dx = \int \sec x \cdot \left(\frac{\sec x + \tan x}{\sec x + \tan x}\right) dx$

$=$

$\int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x}\,dx$

Let $u = \sec x + \tan x$

$du = \sec x \tan x + \sec^2 x\,dx$

$\int \sec x\,dx = \int \frac{1}{u}\,du = \ln|u| + C = \ln|\sec x + \tan x| + C$

[redacted]

47. $\int \sin^3 2x\,dx = \int (\sin^2 2x) \sin 2x\,dx$

$= \int (1 - \cos^2 2x) \sin 2x\,dx$

$u = \cos 2x$

$du = -\sin 2x\,dx$

$= \int (1 - u^2)\,du$

$= u - \frac{u^3}{3} + C$

$= \cos 2x - \frac{\cos^3 2x}{3} + C$

[redacted]

49. $\int 2\sin^2 x\,dx = \int (1 + 2\sin^2 x - 1)dx$

$= \int (1 + \cos 2x)\,dx$

$u = 2x$

$du = 2\,dx$

$= \frac{1}{2}\int (1 + \cos u)\,du$

$= \frac{1}{2}(u + \sin u) + C$

$= x + \frac{\sin 2x}{2} + C$

[redacted]

51. $\int \tan^4 x = \int \tan^3 x\,(\sec^2 x - 1)\,dx$

$= \int (\tan^3 x \sec^2 x - \tan^2 x)\,dx$

$u = \tan x$

$du = \sec^2 x\,dx$

$= \int (u^2 - u)\,du$

$= \frac{u^3}{3} - u + C$

$= \frac{\tan^3 x}{3} - \tan x + x + C$

[redacted]

53. Let $u = y + 1$

$du = dy$

$\int_0^3 \sqrt{y+1}\,dy = \int_1^4 u^{1/2}\,du$

$= \frac{2}{3}u^{3/2}\Big]_1^4$

$= \frac{2}{3}(4)^{3/2} - \frac{2}{3}(1)^{3/2}$

$= \frac{2}{8}(8) - \frac{2}{3} = \frac{14}{3}$

[redacted]

55. Let $u = \tan x$

$du = \sec^2 x\,dx$

$$\int_{-\pi/4}^{0} \tan x \sec^2 x\,dx = \int_{-1}^{0} u\,du$$
$$= \frac{1}{2}u^2\Big]_{-1}^{0}$$
$$= \frac{1}{2}(0) - \frac{1}{2}(-1)^2$$
$$= -\frac{1}{2}$$

[redacted]

57. Let $u = 1 + \theta^{3/2}$

$du = \frac{3}{2}\theta^{1/2}\,d\theta$

$\frac{2}{3}du = \theta^{1/2}\,d\theta$

$$\int_0^1 \frac{10\sqrt{\theta}}{(1+\theta^{3/2})^2}\,d\theta = \frac{2}{3}(10)\int_1^2 u^{-2}\,du$$
$$= -\frac{20}{3}u^{-1}\Big]_1^2$$
$$= -\frac{20}{3}\left(\frac{1}{2} - 1\right)$$
$$= -\frac{20}{3}\left(-\frac{1}{2}\right) = \frac{10}{3}$$

[redacted]

59. Let $u = t^5 + 2t$

$du = (5t^4 + 2)\,dt$

$$\int_0^1 \sqrt{t^5 + 2t}\,(5t^4 + 2)\,dt = \int_0^3 u^{1/2}\,du$$
$$= \frac{2}{3}u^{3/2}\Big]_0^3$$
$$= \frac{2}{3}(3)^{3/2}$$
$$= \frac{2}{3}\sqrt{27} = 2\sqrt{3}$$

[redacted]

61. $\int_0^7 \frac{dx}{x+2}$

$u = x + 2$

$du = dx$

$$\int_0^7 \frac{du}{u} = \ln u\Big|_0^7 = \ln(x+2)\Big|_0^7 = \ln\left(\frac{9}{2}\right) = 1.504$$

[redacted]

63. $\int_1^2 \frac{dt}{t-3}$

$u = t - 3$

$du = dt$

$$\int_1^2 \frac{du}{u} = \ln u\Big|_1^2 = \ln(t-3)\Big|_1^2 = \ln\left(\frac{1}{2}\right) = -0.693$$

[redacted]

65. $\int_{-1}^3 \frac{x\,dx}{x^2+1}$

$u = x^2 + 1$

$du = 2x\,dx$

$$\frac{1}{2}\int_{-1}^3 \frac{du}{u} = \frac{1}{2}\ln u\Big|_{-1}^3 = \frac{1}{2}\ln(x^2+1)\Big|_{-1}^3 = \frac{1}{2}\ln(5) = 0.805$$

[redacted]

67. Let $u = x^4 + 9,\ du = 4x^3\,dx$.

(a) $\int_0^1 \frac{x^3 dx}{\sqrt{x^4+9}} = \int_9^{10} \frac{1}{4}u^{-1/2}\,du = \frac{1}{2}u^{1/2}\Big]_9^{10}$

$= \frac{1}{2}\sqrt{10} - \frac{1}{2}\sqrt{9}$

$= \frac{1}{2}\sqrt{10} - \frac{3}{2} \approx 0.081$

(b) $\int \frac{x^3}{x^4+9}\,dx = \int \frac{1}{4}u^{-1/2}\,du$

$= \frac{1}{2}u^{1/2} + C$

$= \frac{1}{2}\sqrt{x^4+9} + C$

$\int_0^1 \frac{x^3}{x^4+9}\,dx = \frac{1}{2}\sqrt{x^4+9}\Big]_0^1$

$= \frac{1}{2}\sqrt{10} - \frac{1}{2}\sqrt{9}$

$= \frac{1}{2}\sqrt{10} - \frac{3}{2} \approx 0.081$

[redacted]

69. We show that $f'(x) = \tan x$ and $f(3) = 5$, where

$f(x) = \ln\left|\frac{\cos 3}{\cos 3}\right| + 5.$

$f'(x) = \frac{d}{dx}\left(\ln\left|\frac{\cos 3}{\cos x}\right| + 5\right)$

$= \frac{d}{dx}(\ln|\cos 3| - \ln|\cos x| + 5)$

$= -\frac{d}{dx}\ln|\cos x|$

$= -\frac{1}{\cos x}(-\sin x) = \tan x$

$f(3) = \left|\frac{\cos 3}{\cos 3}\right| + 5 = (\ln 1) + 5 = 5$

[redacted]

71. False. The interval of integration should change from $[0, \pi/4]$ to $[0,1]$, resulting in a different numerical answer.

[redacted]

73. D.

[redacted]

75. B. $\int_3^5 F(x-a)\,dx = F(5-a) - F(3-a) = 7$

$\int_{3-a}^{5-a} F(x)\,dx = F(5-a) - F(3-a) = 7$

[redacted]

77. (a) Let $u = x+1$

$du = dx$

$\int \sqrt{x+1}\,dx = \int u^{1/2}du$

$= \frac{2}{3}u^{3/2} + C$

$= \frac{2}{3}(x+1)^{3/2} + C$

Alternatively, $\frac{d}{dx}\left(\frac{2}{3}(x+1)^{3/2} + C\right) = \sqrt{x+1}.$

(b) By Part 1 of the Fundamental Theorem of Calculus, $\frac{dy_1}{dx} = \sqrt{x+1}$ and $\frac{dy_2}{dx} = \sqrt{x+1}$, so both are antiderivatives of $\sqrt{x+1}$.

77. Continued

(c) Using NINT to find the values of y_1 and y_2, we have:

x	0	1	2	3	4
y_1	0	1.219	2.797	4.667	6.787
y_2	−4.667	−3.448	−1.869	0	2.120
$y_1 - y_2$	4.667	4.667	4.667	4.667	4.667

$C = 4\frac{2}{3}$

(d) $C = y_1 - y_2$

$$= \int_0^x \sqrt{x+1}\,dx - \int_3^x \sqrt{x+1}\,dx$$
$$= \int_0^x \sqrt{x+1}\,dx + \int_x^3 \sqrt{x+1}\,dx$$
$$= \int_0^3 \sqrt{x+1}\,dx$$

[redacted]

[redacted]

79. (a) $\int 2\sin x\cos x\,dx = \int 2u\,du = u^2 + C = \sin^2 x + C$

(b) $\int 2\sin x\cos x\,dx = -\int 2u\,du = -u^2 + C = -\cos^2 x + C$

(c) Since $\sin^2 x - (-\cos^2 x) = 1$, the two answers differ by a constant (accounted for in the constant of integration).

[redacted]

81. (a) $\int \frac{dx}{\sqrt{1-x^2}} = \int \frac{\cos u\,du}{\sqrt{1-\sin^2 u}} = \frac{\cos u\,du}{\sqrt{\cos^2 u}} = \int 1du.$

(Note $\cos u > 0$, so $\sqrt{\cos^2 u} = |\cos u| = \cos u$.)

(b) $\int \frac{dx}{\sqrt{1-x^2}} = \int 1du = u + C = \sin^{-1} x + C$

[redacted]

83. (a) $\int_0^{1/2} \frac{\sqrt{x}dx}{\sqrt{1-x}} = \int_{\sin^{-1}\sqrt{0}}^{\sin^{-1}\sqrt{1/2}} \frac{\sin y \bullet 2\sin y\cos y\,dy}{\sqrt{1-\sin^2 y}}$

$$= \int_0^{\pi/4} \frac{2\sin^2 y\cos y\,dy}{\cos y} = \int_0^{\pi/4} 2\sin^2 y\,dy$$

(b) $\int_0^{1/2} \frac{\sqrt{x}dx}{\sqrt{1-x}} = \int_0^{\pi/4} 2\sin^2 y\,dy$

$$= \int_0^{\pi/4} (1-\cos 2y)dy = [y - (1/2)\sin 2y]_0^{\pi/4}$$
$$= (\pi - 2)/4$$

[redacted]

Section 6.3 Antidifferentiation by Parts (pp. 341–349)

Exploration 1 Choosing the Right *u* and *dv*

1. $u = 1 \qquad du = 0$

   $dv = x\cos x \quad v = \int x\cos x\,dx$

   Using 1 for *u* is never a good idea because it places us back where we started.

[redacted]

3. $u = \cos x \quad du = -\sin x$

   $dv = x\,dx \qquad v = \int x\,dx = x^2$

   The selection of $dv = x\,dx$ will place a more difficult integral into $\int v\,du$.

[redacted]

Quick Review 6.3

1. $\frac{dy}{dx} = (x^3)(\cos 2x)(2) + (\sin 2x)(3x^2)$

   $= 2x^3\cos 2x + 3x^2\sin 2x$

[redacted]

3. $\frac{dy}{dx} = \frac{1}{1+(2x)^2} \cdot 2$

   $= \frac{2}{1+4x^2}$

[redacted]

5. $y = \tan^{-1} 3x$

   $\tan y = 3x$

   $x = \frac{1}{3}\tan y$

[redacted]

7. $\int_0^1 \sin \pi x\,dx = -\frac{1}{\pi}\cos \pi x\Big]_0^1$

   $= -\frac{1}{\pi}\cos\pi + \frac{1}{\pi}\cos 0$

   $= -\frac{1}{\pi}(-1) + \frac{1}{\pi} = \frac{2}{\pi}$

[redacted]

9. $\frac{dy}{dx} = x + \sin x$

   $dy = (x + \sin x)dx$

   Integrate both sides.

   $\int dy = \int (x + \sin x)dx$

   $y = \frac{1}{2}x^2 - \cos x + C$

   $y(0) = -1 + C = 2$

   $C = 3$

   $y = \frac{1}{2}x^2 - \cos x + 3$

[redacted]

Section 6.3 Exercises

1. $\int x\sin x\,dx$

   $dv = \sin x\,dx \quad v = \int \sin x\,dx = -\cos x$

   $u = x \qquad du = dx$

   $-x\cos x - \int -\cos x\,dx = -x\cos x + \sin x + C$

[redacted]

3. $\int 3t\, e^{2t} dt$

$dv = e^{2t} dt \qquad v = \int e^{2t} dt = \dfrac{e^{2t}}{2}$

$u = 3t \qquad du = 3\, dt$

$3t\dfrac{e^{2t}}{2} - \int 3\dfrac{e^{2t}}{2} dt = \dfrac{3}{2}te^{2t} - \dfrac{3}{4}e^{2t} + C$

[redacted]

5. $\int x^2 \cos x\, dx$

$dv = \cos x\, dx \qquad v = \int \cos x\, dx = \sin x$

$u = x^2 \qquad du = 2x\, dx$

$x^2 \sin x - \int 2x \sin x\, dx$

$dv = \sin x\, dx \qquad v = \int \sin x\, dx = -\cos x$

$u = 2x \qquad du = 2dx$

$x^2 \sin x + 2x\cos x - \int 2\cos x\, dx$

$= x^2 \sin x + 2x \cos x - 2\sin x + C$

[redacted]

7. $\int 3x^2\, e^{2x} dx$

$dv = e^{2x} dx \qquad v = \int e^{2x} dx = \dfrac{e^{2x}}{2}$

$u = 3x^2 \qquad du = 6x$

$3x^2\dfrac{e^{2x}}{2} - \int 6x\dfrac{e^{2x}}{2} dx = \dfrac{3}{2}x^2e^{2x} - \int 3x\, e^{2x} dx$

$dv = e^{2x} \qquad v = \int e^{2x} dx = \dfrac{e^{2x}}{2}$

$u = 3x \qquad du = 3dx$

$\dfrac{3}{2}x^2e^{2x} - \dfrac{3}{2}xe^{2x} - \int 3\dfrac{e^{2x}}{2} dx = \dfrac{3}{2}x^2e^{2x} - \dfrac{3}{2}xe^{2x} + \dfrac{3}{4}e^{2x} + C$

[redacted]

9. $\int y \ln y\, dy$

$dv = y\, dy \qquad v = \int y\, dy = \dfrac{y^2}{2}$

$u = \ln y \qquad du = \dfrac{1}{y} dy$

$\dfrac{1}{2}y^2 \ln y - \int \dfrac{y^2}{2}\dfrac{1}{y} dy = \dfrac{1}{2}y^2 \ln y - \dfrac{y^2}{4} + C$

[redacted]

11. $\int dy = \int \left((x+2)\sin x\right) dx$

$dv = \sin x\, dx \qquad v = \int \sin x\, dx = -\cos x$

$u = x + 2 \qquad du = dx$

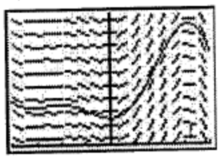
$-(x+2)\cos x - \int -\cos x\, dx = -(x+2)\cos x + \sin x + C$

$2 = -(0+2)\cos(0) + \sin(0) + C$

$2 = -2 + C$

$C = 4$

$y = -(x+2)\cos x + \sin x + 4$



[–4, 4] by [0, 10]

[redacted]

13. $\int du = \int x \sec^2 x\,dx$

$dv = \int \sec^2 x\,dx \qquad v = \int \sec^2 x\,dx = \tan x$

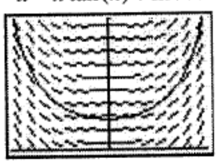
$w = x \qquad dw = dx$

$x \tan x - \int \tan x\,dx = x \tan x + \ln|\cos x| + C$

$1 = 0 \tan(0) + \ln|\cos(0)| + C$

$C = 1$

$u = x\tan(x) + \ln|\cos(x)| + 1$



[−1.2, 1.2] by [0, 3]

[redacted]

15. $\int dy = \int x\sqrt{x-1}\,dx$

$dv = (x-1)^{1/2} \qquad v = \int (x-1)^{1/2}dx = \frac{2}{3}(x-1)^{3/2}$

$u = x \qquad du = dx$

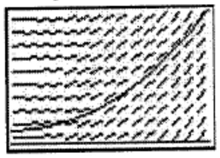
$\frac{2}{3}x(x-1)^{3/2} - \int \frac{2}{3}(x-1)^{3/2}dx$

$= \frac{2}{3}x(x-1)^{3/2} - \frac{4}{15}(x-1)^{5/2} + C$

$2 = \frac{2}{3}(1)(1-1)^{3/2} - \frac{4}{15}(1-1)^{5/2} + C$

$C = 2$

$y = \frac{2}{3}x(x-1)^{3/2} - \frac{4}{15}(x-1)^{5/2} + 2$



[1, 5] by [0, 20]

[redacted]

17. $\int e^x \sin x\,dx$

$dv = e^x\,dx \qquad v = \int e^x\,dx = e^x$

$u = \sin x \qquad du = \cos x\,dx$

$e^x \sin x - \int e^x \cos x d$

$dv = e^x\,dx \qquad v = \int e^x\,dx = e^x$

$u = \cos x \qquad du = -\sin x\,dx$

$\int e^x \sin x\,dx = e^x \sin x - (e^x \cos x - \int -e^x \sin x\,dx)$

$\int e^x \sin x\,dx = \frac{e^x}{2}(\sin x - \cos x) + C$

[redacted]

19. $\int e^x \cos 2x \, dx$

$dv = \cos 2x \, dx \quad v = \int \cos 2x \, dx = 2 \sin 2x$

$u = e^x \qquad du = e^x dx$

$2e^x \sin 2x - \int 2 \sin 2x \, e^x dx$

$dv = 2 \sin 2x \qquad v = \int 2 \sin 2x \, dx = -4 \cos 2x$

$u = e^x \qquad du = e^x dx$

$\int e^x \cos 2x \, dx$

$= 2e^x \sin 2x - (-4e^x \cos 2x - \int -e^x dx \;4 \cos 2x \, dx)$

$\int e^x \cos 2x \, dx = \frac{e^x}{5}(2 \sin 2x + \cos 2x) + C$

[redacted]

21. Use tabular integration with $f(x) = x^4$ and $g(x) = e^{-x}$.

$f(x)$ and its derivatives		$g(x)$ and its integrals
x^4	(+)	e^{-x}
$4x^3$	(−)	$-e^{-x}$
$12x^2$	(+)	e^{-x}
$24x$	(−)	$-e^{-x}$
24	(+)	e^{-x}
0		$-e^{-x}$

$\int x^4 e^{-x} \, dx$

$= -x^4 e^{-x} - 4x^3 e^{-x} - 12x^2 e^{-x} - 24xe^{-x} - 24e^{-x} + C$

$= -(x^4 + 4x^3 + 12x^2 + 24x + 24)e^{-x} + C$

[redacted]

23. Use tabular integration with $f(x) = x^3$ and $g(x) = e^{-2x}$.

$f(x)$ and its derivatives		$g(x)$ and its integrals
x^3	(+)	e^{-2x}
$3x^2$	(−)	$-\frac{1}{2} e^{-2x}$
$6x$	(+)	$\frac{1}{4} e^{-2x}$
6	(−)	$-\frac{1}{8} e^{-2x}$
0		$\frac{1}{16} e^{-2x}$

$\int x^3 e^{-2x} \, dx$

$= -\frac{1}{2} x^3 e^{-2x} - \frac{3}{4} x^2 e^{-2x} - \frac{3}{4} xe^{-2x} - \frac{3}{8} e^{-2x} + C$

$= -\left(\frac{x^3}{2} + \frac{3x^2}{4} + \frac{3x}{4} + \frac{3}{8}\right) e^{-2x} + C$

[redacted]

25. Use tabular integration with $f(x) = x^2$ and $g(x) = \sin 2x$.

$f(x)$ and its derivatives		$g(x)$ and its integrals
x^2	(+)	$\sin 2x$
$2x$	(−)	$-\frac{1}{2}\cos 2x$
2	(+)	$-\frac{1}{4}\sin 2x$
0		$\frac{1}{8}\cos 2x$

$$\int x^2 \sin 2x\,dx = -\frac{1}{2}x^2\cos 2x + \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + C$$

$$= \left(\frac{1-2x^2}{4}\right)\cos 2x + \frac{x}{2}\sin 2x + C$$

$$\int_0^{\pi/2} x^2 \sin 2x\,dx = \left[\left(\frac{1-2x^2}{4}\right)\cos 2x + \frac{x}{2}\sin 2x\right]_0^{\pi/2}$$

$$= \left(\frac{1-2\left(\frac{\pi}{2}\right)^2}{4}\right)(-1) + 0 - \left(\frac{1}{4}\right)(1) - 0$$

$$= \frac{\pi^2}{8} - \frac{1}{2} \approx 0.734$$

Check: $\text{NINT}\left(x^2\sin 2x, x, 0, \frac{\pi}{2}\right) \approx 0.734$

[redacted]

[redacted]

27. Let $u = e^{2x}$ $\quad dv = \cos 3x\,dx$

$du = 2e^{2x}\,dx \quad v = \frac{1}{3}\sin 3x$

$$\int e^{2x}\cos 3x\,dx = (e^{2x})\left(\frac{1}{3}\sin 3x\right) - \int\left(\frac{1}{3}\sin 3x\right)(2e^{2x}\,dx)$$

$$= \frac{1}{3}e^{2x}\sin 3x - \frac{2}{3}\int e^{2x}\sin 3x\,dx$$

Let $u = e^{2x}$ $\quad dv = \sin 3x\,dx$

$du = 2e^{2x}\,dx \quad v = -\frac{1}{3}\cos 3x$

$$\int e^{2x}\cos 3x\,dx = \frac{1}{3}e^{2x}\sin 3x - \frac{2}{3}\left[(e^{2x})\left(-\frac{1}{3}\cos 3x\right) - \int\left(-\frac{1}{3}\cos 3x\right)(2e^{2x}\,dx)\right]$$

$$= \frac{1}{9}e^{2x}(3\sin 3x + 2\cos 3x) - \frac{4}{9}\int e^{2x}\cos 3x\,dx$$

$$\frac{13}{9}\int e^{2x}\cos 3x\,dx = \frac{1}{9}e^{2x}(3\sin 3x + 2\cos 3x)$$

$$\int e^{2x}\cos 3x\,dx = \frac{1}{13}e^{2x}(3\sin 3x + 2\cos 3x)$$

$$\int_{-2}^{3} e^{2x}\cos 3x\,dx = \left[\frac{1}{13}e^{2x}(3\sin 3x + 2\cos 3x)\right]_{-2}^{3}$$

$$= \frac{1}{13}[e^6(3\sin 9 + 2\cos 9) - e^{-4}(3\sin(-6) + 2\cos(-6))]$$

$$= \frac{1}{13}[e^6(2\cos 9 + 3\sin 9) - e^{-4}(2\cos 6 - 3\sin 6)]$$

$$\approx -18.186$$

Check: $\text{NINT}\left(e^{2x}\cos 3x, x, -2, 3\right) \approx -18.186$

[redacted]

[redacted]

29. $y = \int x^2 e^{4x}\,dx$

Let $u = x^2 \qquad dv = e^{4x}dx$

$du = 2x\,dx \qquad v = \frac{1}{4}e^{4x}$

$$y = (x^2)\left(\frac{1}{4}e^{4x}\right) - \int\left(\frac{1}{4}e^{4x}\right)(2x\,dx)$$

$$= \frac{1}{4}x^2e^{4x} - \frac{1}{2}\int xe^{4x}dx$$

Let $u = x \qquad dv = e^{4x}\,dx$

$du = dx \qquad v = \frac{1}{4}e^{4x}$

$$y = \frac{1}{4}x^2e^{4x} - \frac{1}{2}\left[(x)\left(\frac{1}{4}e^{4x}\right) - \int\left(\frac{1}{4}e^{4x}\right)dx\right]$$

$$y = \frac{1}{4}x^2e^{4x} - \frac{1}{8}xe^{4x} + \frac{1}{32}e^{4x} + C$$

$$y = \left(\frac{x^2}{4} - \frac{x}{8} + \frac{1}{32}\right)e^{4x} + C$$

[redacted]

31. $y = \int \theta \sec^{-1}\theta\,d\theta$

Let $u = \sec^{-1}\theta \qquad dv = \theta\,d\theta$

$du = \frac{1}{\theta\sqrt{\theta^2-1}}du \qquad v = \frac{1}{2}\theta^2$

Note that we are told $\theta > 1$, so no absolute value is needed in the expression for du.

$$y = (\sec^{-1}\theta)\left(\frac{1}{2}\theta^2\right) - \int\left(\frac{1}{2}\theta^2\right)\left(\frac{1}{\theta\sqrt{\theta^2-1}}d\theta\right)$$

$$y = \frac{\theta^2}{2}\sec^{-1}\theta - \frac{1}{4}\int\frac{2|\theta|\,d\theta}{\sqrt{\theta^2-1}}$$

Let $w = \theta^2 - 1,\ dw = 2\theta\,d\theta$

$$y = \frac{\theta^2}{2}\sec^{-1}\theta - \frac{1}{4}\int w^{-1/2}dw$$

$$y = \frac{\theta^2}{2}\sec^{-1}\theta - \frac{1}{2}w^{1/2} + C$$

$$y = \frac{\theta^2}{2}\sec^{-1}\theta - \frac{1}{2}\sqrt{\theta^2-1} + C$$

[redacted]

33. Let $u = x \qquad dv = \sin x\,dx$

$du = dx \qquad v = -\cos x$

$$\int x\sin x\,dx = -x\cos x + \int\cos x\,dx$$
$$= -x\cos x + \sin x + C$$

(a) $\int_0^\pi |x\sin x|\,dx = \int_0^\pi x\sin x\,dx$

$$= \left[-x\cos x + \sin x\right]_0^\pi$$
$$= -\pi(-1) + 0 + 0(1) - 0$$
$$= \pi$$

33. Continued

(b) $\int_{\pi}^{2\pi} |x \sin x| \, dx = -\int_{\pi}^{2\pi} x \sin x \, dx$

$= \left[x \cos x - \sin x \right]_{\pi}^{2\pi}$

$= 2\pi(1) - 0 - \pi(-1) + 0$

$= 3\pi$

(c) $\int_{\pi}^{2\pi} |x \sin x| \, dx = \int_{0}^{\pi} |x \sin x| \, dx + \int_{\pi}^{2\pi} |x \sin x| \, dx$

$= \pi + 3\pi = 4\pi$

[redacted]

35. First, we evaluate $\int e^{-t} \cos t \, dt$.

Let $u = e^{-t}$ $\quad dv = \cos t \, dt$

$du = -e^{-t} \, dt$ $\quad v = \sin t$

$\int e^{-t} \cos t \, dt = e^{-t} \sin t + \int \sin t \; e^{-t} \, dt$

Let $u = e^{-t}$ $\quad dv = \sin t \, dt$

$du = -e^{-t} \, dt$ $\quad v = -\cos t$

$\int e^{-t} \cos t \, dt = e^{-t} \sin t - e^{-t} \cos t - \int e^{-t} \cos t \, dt$

$2\int e^{-t} \cos t \, dt = e^{-t}(\sin t - \cos t) + C$

$\int e^{-t} \cos t \, dt = \frac{1}{2} e^{-t}(\sin t - \cos t) + C$

Now we find the average value of $y = 2e^{-t} \cos t$ for $0 \le t \le 2\pi$.

Average value $= \frac{1}{2\pi} \int_0^{2\pi} 2e^{-t} \cos t \, dt$

$= \frac{1}{\pi} \int_0^{2\pi} e^{-t} \cos t \, dt$

$= \frac{1}{2\pi} e^{-t}(\sin t - \cos t) \Big]_0^{2\pi}$

$= \frac{1}{2\pi} \left[e^{-2\pi}(-1) - e^{0}(-1) \right]$

$= \frac{1 - e^{-2\pi}}{2\pi} \approx 0.159$

[redacted]

37. True. Use parts, letting $u = x^2$, $dv = g(x)dx$, and $v = f(x)$.

[redacted]

39. B. $\int x \sin(5x) \, dx$

$dv = \sin(5x) \, dx$ $\quad v = \int \sin(5x) \, dx = -\frac{1}{5} \cos 5x$

$u = x$ $\quad du = dx$

$-\frac{1}{5} x \cos(5x) - \int -\frac{1}{5} \cos(5x) \, dx = -\frac{1}{5} x \cos x$

$+ \frac{1}{25} \sin(5x)$

[redacted]

41. C. $\int dy = \int 4x \ln x \, dx$

$dv = 4x \, dx$ $\quad v = \int 4x \, dx = 2x^2$

$u = \ln x$ $\quad du = \frac{1}{x} \, dx$

$2x^2 \ln x - \int 2x^2 \frac{1}{x} \, dx = 2x^2 \ln x - x^2 + C$

[redacted]

[redacted]

43. Let $w=\sqrt{x}$. Then $dw=\dfrac{dx}{2\sqrt{x}}$, so $dx=2\sqrt{x}\ dw=2w\,dw$.

$$\int \sin \sqrt{x}\ dx=\int (\sin w)(2w\ dw)=2\int w \sin w\,dw$$

Let $u=w$ $\quad dv=\sin w\,dw$

$du=dw$ $\quad v=-\cos w$

$$\int w \sin w\,dw=-w\cos w+\int \cos w\,dw$$
$$=-w\cos w+\sin w+C$$

$$\int \sin\sqrt{x}\ dx=2\int w \sin w\,dw$$
$$=-2w\cos w+2\sin w+C$$
$$=-2\sqrt{x}\cos\sqrt{x}+2\sin\sqrt{x}+C$$

[redacted]

[redacted]

45. Let $w=x^2$. Then $dw=2x\,dx$.

$$\int x^7 e^{x^2}dx=\int (x^2)^3 e^{x^2}x\,dx=\frac{1}{2}\int w^3\ e^w\ dw.$$

Use tabular integration with $f(x)=w^3$ and $g(w)=e^w$.

$f(w)$ and its derivatives		$g(w)$ and its integrals
w^3	(+)	e^w
$3w^2$	(−)	e^w
$6w$	(+)	e^w
6	(−)	e^w
0		e^w

$$\int w^3 e^w\,dw=w^3e^w-3w^2e^w+6we^w-6e^w+C$$
$$=(w^3-3w^2+6w-6)e^w+C$$

$$\int x^7e^{x^2}\ dx=\frac{1}{2}\int w^3\ e^w\ dw$$
$$=\frac{1}{2}(w^3-3w^2+6w-6)e^w+C$$
$$=\frac{(x^6-3x^4+6x^2-6)e^{x^2}}{2}+C$$

[redacted]

47. Let $u=x^n$ $\quad dv=\cos x\ dx$

$du=nx^{n-1}dx$ $\quad v=\sin x$

$$\int x^n\cos x\ dx=x^n\sin x-\int(\sin x)(nx^{n-1}dx)$$
$$=x^n\sin x-n\int x^{n-1}\sin x\ dx$$

[redacted]

49. Let $u = x^n \qquad dv = e^{ax}\,dx$

$du = nx^{n-1}\,dx \qquad v = \frac{1}{a}e^{ax}$

$$\int x^n e^{ax}\,dx = (x^n)\left(\frac{1}{a}e^{ax}\right) - \int\left(\frac{1}{a}e^{ax}\right)(nx^{n-1}\,dx)$$

$$= \frac{x^n e^{ax}}{a} - \frac{n}{a}\int x^{n-1}e^{ax}\,dx,\ a \neq 0$$

[redacted]

51. (a) Let $y = f^{-1}(x)$. Then $x = f(y)$, so $dx = f'(y)\,dy$.

Hence, $\int f^{-1}(x)\,dx = \int (y)[f'(y)\,dy] = \int y\,f'(y)\,dy$

(b) Let $u = y \qquad dv = f'(y)dy$

$du = dy \qquad v = f(y)$

$$\int y\,f'(y)\,dy = y\,f(y) - \int f(y)\,dy$$

$$= f^{-1}(x)(x) - \int f(y)\,dy$$

Hence, $\int f^{-1}(x)\,dx = \int y\,f'(y)\,dy$

$$= x\,f^{-1}(x) - \int f(y)\,dy.$$

[redacted]

53. (a) Using $y = f^{-1}(x) = \sin^{-1} x$ and $f(y) = \sin y$,

$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$, we have:

$$\int \sin^{-1}x\,dx = x\sin^{-1}x - \int \sin y\,dy$$

$$= x\sin^{-1}x + \cos y + C$$

$$= x\sin^{-1}x + \cos(\sin^{-1}x) + C$$

(b) $\int \sin^{-1}x\,dx = x\sin^{-1}x - \int x\left(\frac{d}{dx}\sin^{-1}x\right)dx$

$$= x\sin^{-1}x - \int x\frac{1}{\sqrt{1-x^2}}dx$$

$u = 1 - x^2,\ du = -2x\,dx$

$$= x\sin^{-1}x + \frac{1}{2}\int u^{-1/2}du$$

$$= x\sin^{-1}x + u^{1/2} + C$$

$$= x\sin^{-1}x + \sqrt{1-x^2} + C$$

(c) $\cos(\sin^{-1}x) = \sqrt{1-x^2}$

[redacted]

55. (a) Using $y = f^{-1}(x) = \cos^{-1}x$ and $f(y) = \cos x,\ 0 \le x \le \pi$, we have:

$$\int \cos^{-1}x\,dx = \cos^{-1}x - \int \cos y\,dy$$

$$= x\cos^{-1}x - \sin y + C$$

$$= x\cos^{-1}x - \sin(\cos^{-1}x) + C$$

(b) $\int \cos^{-1}x\,dx = x\cos^{-1}x - \int x\left(\frac{d}{dx}\cos^{-1}x\right)dx$

$$= x\cos^{-1}x - \int x\left(-\frac{1}{\sqrt{1-x^2}}\right)dx$$

$u = 1 - x^2,\ du = -2x\,dx$

$$= x\cos^{-1}x - \frac{1}{2}\int u^{-1/2}\,du$$

$$= x\cos^{-1}x - u^{1/2} + C$$

$$= x\cos^{-1}x - \sqrt{1-x^2} + C$$

(c) $\sin(\cos^{-1}x) = \sqrt{1-x^2}$

[redacted]

Quick Quiz Section 6.1–6.3

1. E.

2. C.

3. A. $\int xe^{2x}dx$

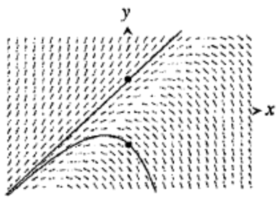
$dv = e^{2x}dx \qquad v = \int e^{2x}dx = \frac{e^{2x}}{2}$

$u = x \qquad du = dx$

$\frac{xe^{2x}}{2} - \int \frac{e^{2x}}{2}dx$

$= \frac{xe^{2x}}{2} - \frac{e^{2x}}{4} + C$

4. (a)



(b) Let $\frac{dy}{dx} = 2$ and $y = 2x + b$ in the differential equation:

$2 = 2(2x + b) - 4x$
$2 = 2b$
$b = 1$

(c) First, note that $\frac{dy}{dx} = 2(0) - 4(0) = 0$ at the point (0, 0).

Also, $\frac{d^2y}{dx^2} = \frac{d}{dx}(2y - 4x) = 2\frac{dy}{dx} - 4$, which is –4 at the point (0, 0).

By the Second Dervative test, g has a local maximum at (0, 0).

Section 6.4 Exponential Growth and Decay (pp.350–361)

Exploration 1 Choosing a Convenient Base

1. $h = \frac{1}{t} = \frac{1}{5}$. h is the reciprocal to the doubling period.

2. $3 = 2^{ht}$

$\frac{5\log 3}{\log 2} = ht$

$\frac{5\log 3}{\log 2} = t = 7.925$ years.

3. $h = \frac{1}{t} = \frac{1}{10}$. h is the reciprocal to the tripling period.

4. $2 = 3^{ht}$

$\frac{\log 2}{\log 3} = ht$

$\frac{10\log 2}{\log 3} = t = 6.3093$ years.

5. $h = \frac{1}{t} = \frac{1}{15}$. h is the reciprocal to the half life.

6. $.10 = \left(\frac{1}{2}\right)^{ht}$

$\frac{\log(.10)}{\log\left(\frac{1}{2}\right)} = ht$

$\frac{15\log(.10)}{\log\left(\frac{1}{2}\right)} = t = 49.83$ years.

Quick Review 6.4

1. $a = e^b$

2. $c = \ln d$

3. $\ln(x+3) = 2$
$x + 3 = e^2$
$x = e^2 - 3$

4. $100e^{2x} = 600$
$e^{2x} = 6$
$2x = \ln 6$
$x = \frac{1}{2}\ln 6$

5. $0.85^x = 2.5$
$\ln 0.85^x = \ln 2.5$
$x \ln 0.85 = \ln 2.5$
$x = \frac{\ln 2.5}{\ln 0.85} \approx -5.638$

6. $2^{k+1} = 3^k$
$\ln 2^{k+1} = \ln 3^k$
$(k+1)\ln 2 = k\ln 3$
$\ln 2 = k(\ln 3 - \ln 2)$
$k = \frac{\ln 2}{\ln 3 - \ln 2} \approx 1.710$