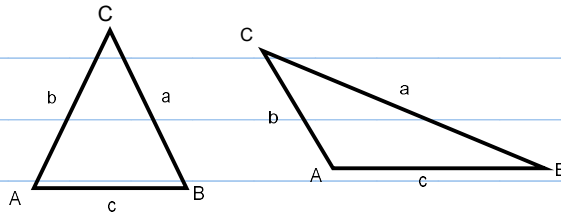


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Objective: use the law of sines to:

- *Solve for unknowns in oblique triangles.
- *Use the law of sines to model and solve real-life problems.
- * Find areas of oblique triangles

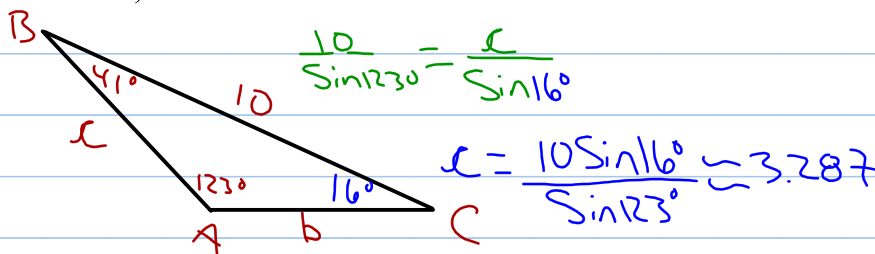
Section 6.1 Law of Sines



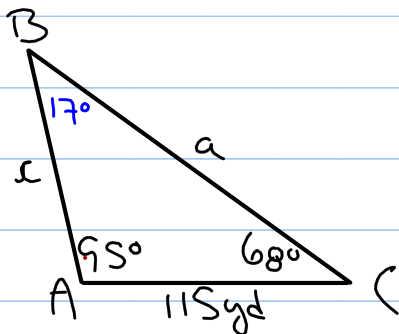
Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \text{ or } \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Example 1. Given $A = 123^\circ$, $B = 41^\circ$, and $a = 10$ inches, find c .



Example 2. A triangular plot of land has interior angles $A = 95^\circ$ and $C = 68^\circ$. If the side between these angles is 115 yards long, what is the length of the other two sides?



$$\frac{a}{\sin 95^\circ} = \frac{115}{\sin 17^\circ}$$

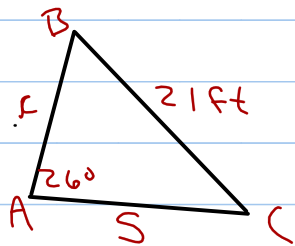
$$a = \frac{115 \sin 95^\circ}{\sin 17^\circ} = 391.838 \text{ yd.}$$

$$\frac{c}{\sin 68^\circ} = \frac{115}{\sin 17^\circ}$$

$$c = \frac{115 \sin 68^\circ}{\sin 17^\circ} = 364.694 \text{ yd.}$$

Example 3. Given $A = 26^\circ$, $b = 5$ feet, and $a = 21$ feet, find the other side and the other two angles.

Check for two solutions when given Angle-Side-Side



$$\frac{\sin B}{5} = \frac{\sin 26^\circ}{21}$$

$$\sin B = \frac{5 \sin 26^\circ}{21}$$

$$\sin B \approx 0.1044$$

$$B = \sin^{-1}(0.1044 \dots)$$

$$C = 148.009^\circ \quad B_1 \approx 5.991^\circ$$

$$\frac{21}{\sin 26^\circ} = \frac{c}{\sin 148.009^\circ}$$

$$c = \frac{21 \sin 148.009^\circ}{\sin 26^\circ}$$

$$c \approx 25.379 \text{ ft}$$

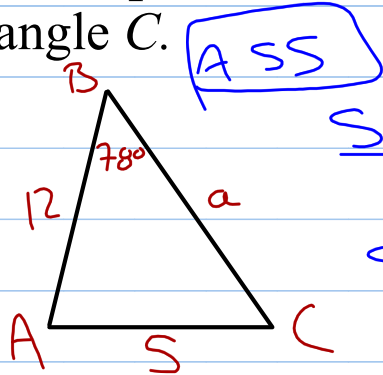
$$B_2 \approx 180 - 5.991^\circ \approx 174.009^\circ$$

$$\sin 26^\circ + \sin 174.009^\circ > 180^\circ$$

B_2 is not possible!

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Example 4. Given $B = 78^\circ$, $c = 12$, and $b = 5$, find angle C.



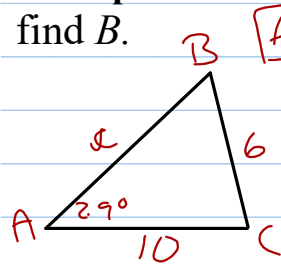
$$\frac{\sin 78^\circ}{5} = \frac{\sin C}{12}$$

$$\sin C = \frac{12 \sin 78^\circ}{5}$$

$$\sin C = 2.348$$

\therefore there is no such Δ !

Example 5. Given $A = 29^\circ$, $a = 6$, and $b = 10$, find B.



$$\frac{\sin 29^\circ}{6} = \frac{\sin B}{10}$$

$$B = 53.903^\circ$$

$$B_1 = 53.903^\circ$$

$$C_1 = 97.097^\circ$$

$$c_1 = 12.281$$

$$B_2 = 180 - 53.903^\circ = 126.097^\circ$$

$$C_2 = 24.903^\circ$$

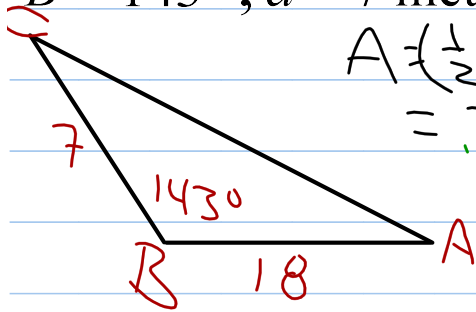
$$c_2 = 5.211$$

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From the law of sines, the area of an oblique triangle is:

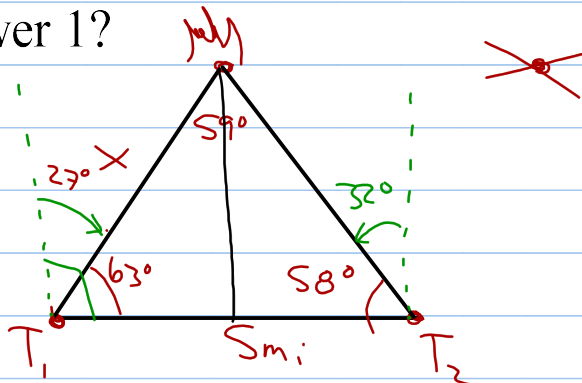
$$\text{Area} = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}cb \sin A = \frac{1}{2}ab \sin C = \frac{1}{2}ac \sin B$$

Example 6. Find the area of the triangle in which $B = 143^\circ$, $a = 7$ meters and $c = 18$ meters.



$$\begin{aligned} A &= \left(\frac{1}{2}\right)(7)(18) \sin 143^\circ \\ &= 37.914 \text{ m}^2 \end{aligned}$$

Example 7. Two fire ranger towers lie on the east-west line and are 5 miles apart. There is a fire with a bearing of $N27^\circ E$ from tower 1 and $N32^\circ W$ from tower 2. How far is the fire from tower 1?



$$\frac{x}{\sin 58^\circ} = \frac{5}{\sin 59^\circ}$$

$$x = 4.947 \text{ mi}$$