Section 6.2 Law of Cosines

In an oblique triangle the law of cosines states:

$$a^2 = b^2 + c^2 - 2bc\cos A \quad or \quad \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

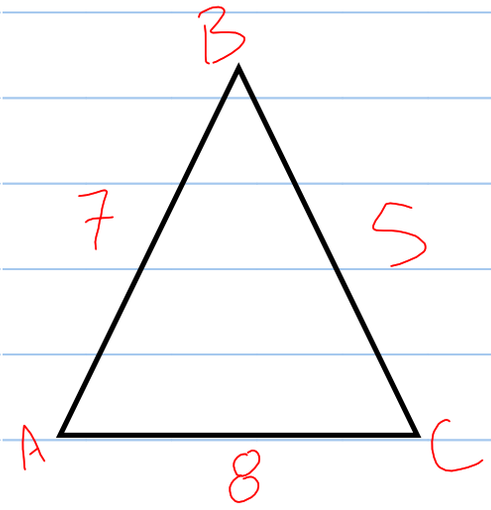
note ∡ means side a starts the equation

$$b^2 = a^2 + c^2 - 2ac\cos B \quad or \quad \cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$c^2 = a^2 + b^2 - 2ab\cos C \quad or \quad \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Example 1. Given $a = 5$, $b = 8$, and $c = 7$, find all three of the angles.

Always find the largest angle first.



(largest ∠ is always opposite the largest side.)

So ∠B is the largest ∠.

$8^2 = 7^2 + 5^2 - 2(7)(5)\cos B$

$8^2 - 7^2 - 5^2 = -2(7)(5)\cos B$

$\frac{8^2 - 7^2 - 5^2}{-2(7)(5)} = \cos B$

$B = \cos^{-1}\left(\frac{8^2 - 7^2 - 5^2}{-2(7)(5)}\right) = 81.787°$

after you find the largest ∠ using the law of cosines, use the law of sines to find the rest.

$\frac{\sin(81.787°)}{8} = \frac{\sin C}{7} \rightarrow \frac{7\sin(81.787°)}{8} = \sin C$

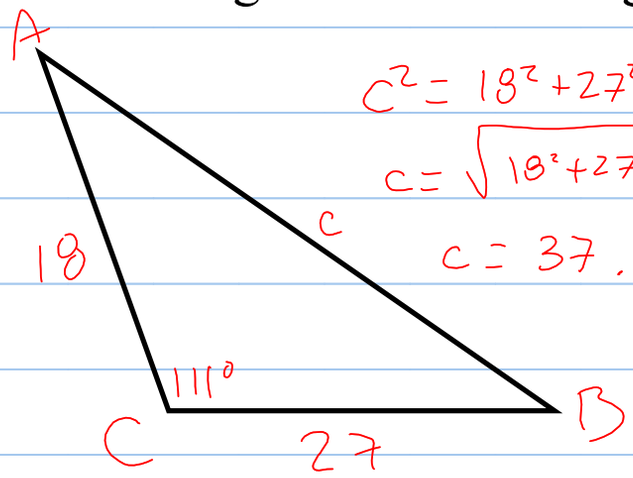
$\rightarrow C = \sin^{-1}\left(\frac{7\sin(81.787°)}{8}\right) = 60.000°$

Now $\angle A = 180 - 60.000° - 81.787°$

$\therefore \angle A = 38.213°$

$\therefore B = 81.787°, C = 60.000°, A = 38.213°$

Example 2. Given C = 111^0, a = 27, and b = 18, find the remaining side and two angles.

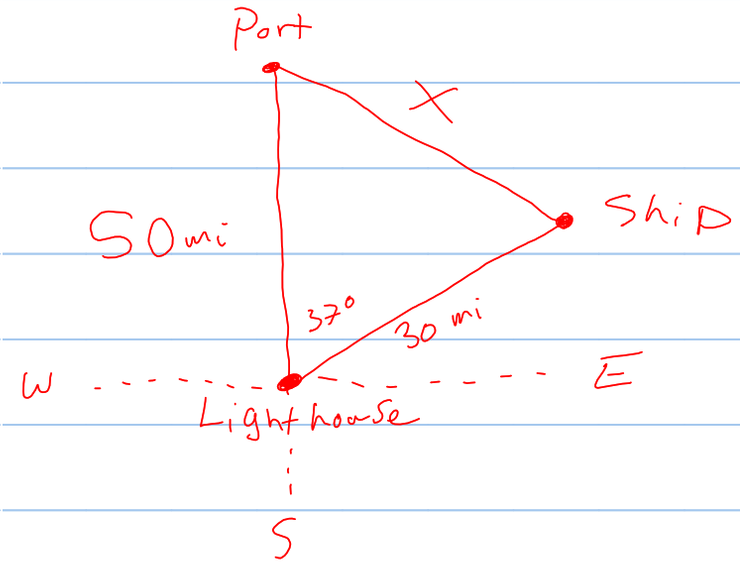


$c^2 = 18^2 + 27^2 - 2(18)(27)\cos(111°)$

$c = \sqrt{18^2 + 27^2 - 2(18)(27)\cos(111°)}$

$c = 37.435$

Example 3. A port is 50 miles due north of a lighthouse. A ship is 30 miles from the lighthouse at a bearing of N37^{0}E. How far is the ship from the port?



type in calculator

$x^2 = 50^2 + 30^2 - 2(50)(30)\cos(37°)$

$x = \sqrt{50^2 + 30^2 - 2(50)(30)\cos(37°)}$

$x = 31.687$

Heron's Area Formula

Calculator to save s

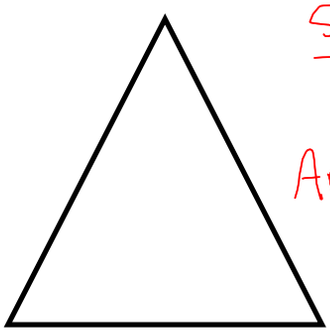
$$Area = \sqrt{s(s-a)(s-b)(s-c)}$$

$$where\ s = \frac{(a+b+c)}{2}$$

if you enter $\frac{(a+b+c)}{2}$ in calculator then press enter you can enter

$\sqrt{}$ [2nd] [(-)] ([2nd] [(-)] − a)([2nd] [(-)] − b)([2nd] [(-)] − c

Example 4. Find the area of the triangular region with sides 50 feet, 58 feet, and 69 feet.



$\frac{50+58+69}{2} = s = 88.5$

$\text{Area} = \sqrt{88.5(88.5-50)(88.5-58)(88.5-69)}$

$\text{Area} = 1423.538\ ft^2$