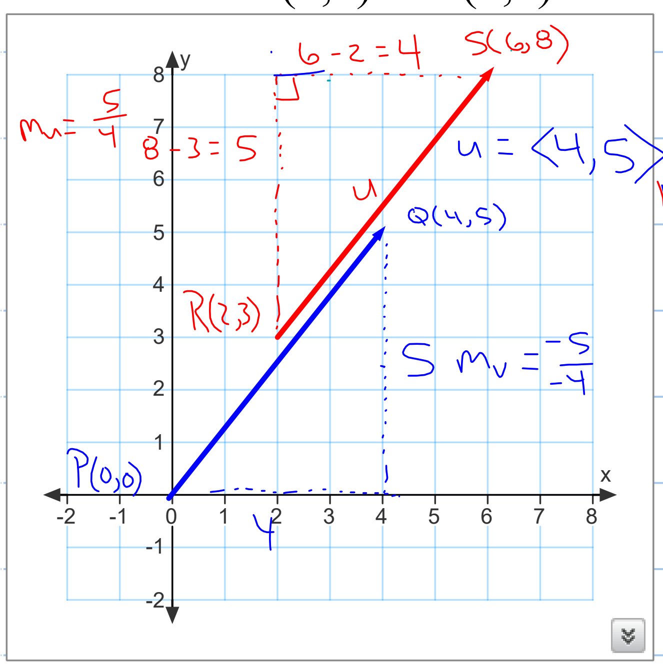
Section 6.3 Vectors in the Plane

Def: Vector - A directed line segment that has both magnitude and direction.

Example 1. Let ***v*** be the vector from P(0,0) to Q(4,5) and ***u*** be the vector from R(2,3) to S(6,8). Show that ***v*** = ***u***.



magnitude of u = $||u||$

$u^2 = 5^2+4^2$

$||u|| = \sqrt{5^2+4^2} = \sqrt{41}$

Magnitude of v = $||v||$

$||v|| = \sqrt{5^2+4^2} = \sqrt{41}$

Component Form of a Vector

Given the vector with initial point $P(P_x, P_y)$ and terminal point $Q(Q_x, Q_y)$

the component form is: $\langle \quad \rangle$

$$\overrightarrow{PQ} = \langle Q_x - P_x, Q_y - P_y \rangle = \langle v_x, v_y \rangle = v$$

$\overleftarrow{QP}$

v_x is the x component of v

v_y is the y component of v

The magnitude of v is:

$$\|v\| = \sqrt{(Q_x - P_x)^2 + (Q_y - P_y)^2} = \sqrt{v_x^2 + v_y^2}$$

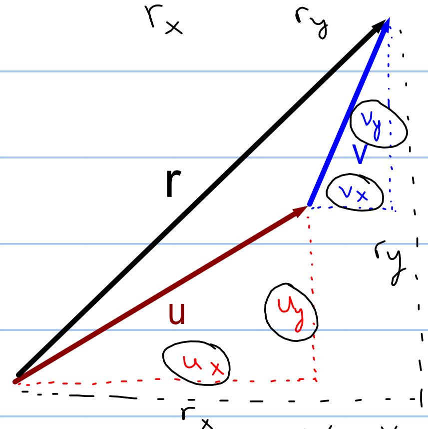
Example 2. Write the component form of vector v with initial point (4,-7) and terminal point (-1,5) and find the magnitude.

$v = \langle -1-4, 5-(-7) \rangle = \langle -5, 12 \rangle$

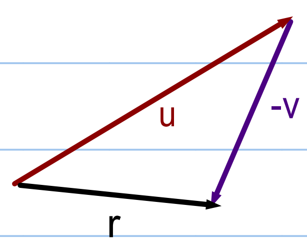
$\|v\| = \sqrt{(-5)^2 + (12)^2} = \sqrt{169} = 13$

Let $u = \langle u_x, u_y \rangle$ and $v = \langle v_x, v_y \rangle$ be vectors and k be a real number called a scalar. Then,

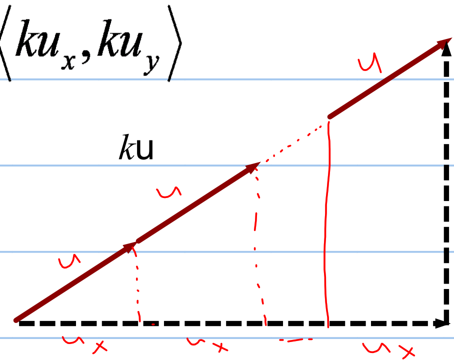
$$u + v = \langle \underbrace{u_x + v_x}_{r_x}, \underbrace{u_y + v_y}_{r_y} \rangle$$



$$u - v = u + (-v) = \langle \underbrace{\overset{u_x - v_x}{u_x + (-v_x)}}_{r_x}, \underbrace{\overset{u_y - v_y}{u_y + (-v_y)}}_{r_y} \rangle$$



$$ku = k\langle u_x, u_y \rangle = \langle ku_x, ku_y \rangle$$



Example 3. Let $u = \langle -5, 2 \rangle$ and $v = \langle 6, -3 \rangle$ find the following.

a) $4u = 4\langle -5, 2 \rangle$

$= \langle -20, 8 \rangle$

b) $u + v = \langle -5+6, 2+(-3) \rangle$

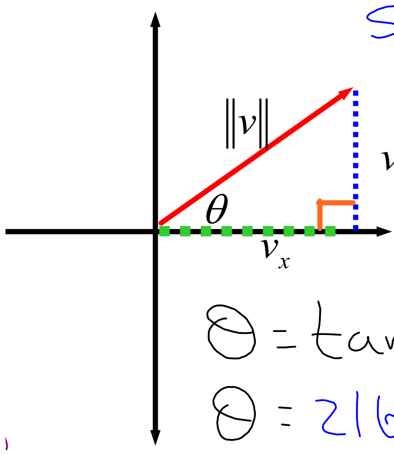
$= \langle 1, -1 \rangle$

c) $2u - v = 2\langle -5, 2 \rangle - \langle 6, -3 \rangle$

$= \langle -10, 4 \rangle + \langle -6, 3 \rangle$

$= \langle -16, 7 \rangle$

Direction Angle of a vector - The positive angle created between the positive x-axis and the vector.



$\sin\theta = \frac{v_y}{\|v\|}$

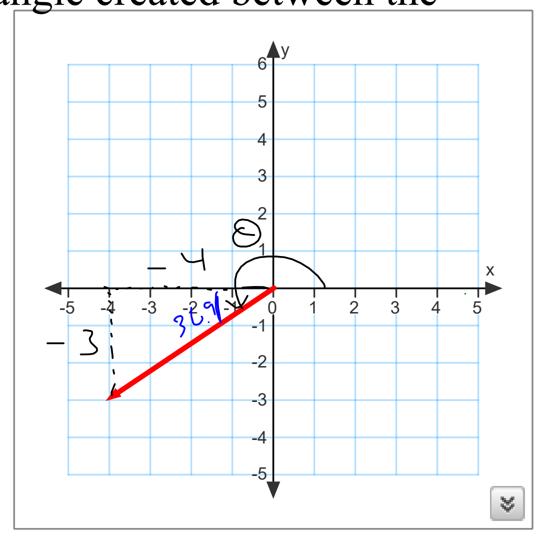
$v_y = \|v\| \sin\theta$

$\cos\theta = \frac{v_x}{\|v\|}$

$v_x = \|v\| \cos\theta$

$\theta = \tan^{-1}\left(\frac{-3}{-4}\right)$

$\theta = 216.9°$



$$v = \langle v_x, v_y \rangle = \langle \|v\|\cos\theta, \|v\|\sin\theta \rangle \quad \textit{where } \tan\theta = \frac{v_y}{v_x}$$

$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$

Example 4. Find the component form of *v* given its magnitude and direction angle. $\|v\| = 3 \; and \; \theta = 45^0$

$\vec{v} = \langle 3\cos 45°, 3\sin 45° \rangle = \langle \frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2} \rangle$

Linear Combination of a vector - Put the vector in component form

$v = \langle v_x, v_y \rangle$, then write as a linear combination with i assigned to v_x

and j assigned to v_y. $v = v_x \mathrm{i} + v_y \mathrm{j}$

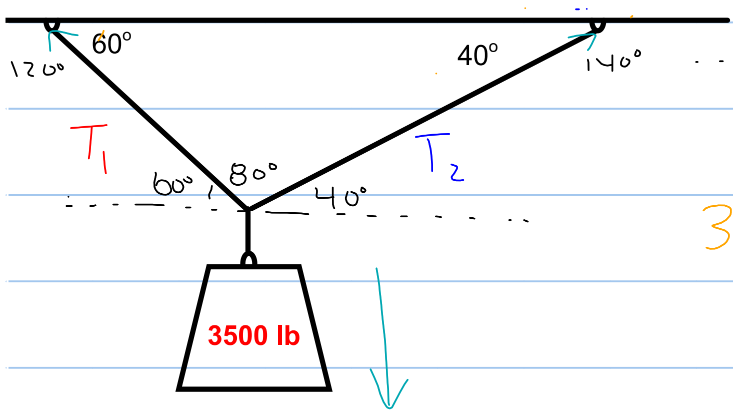
Example 5. Let r be a vector with initial point (3,-2) and terminal point (10,7). Write r as a linear combination of i and j.

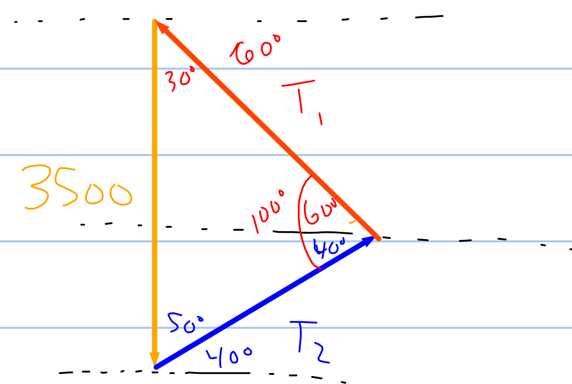
$\vec{r} = \langle 10-3, 7-(-2) \rangle$

$\vec{r} = \langle 7, 9 \rangle$

so $\vec{r} = 7i + 9j$

Example 6. Two cables hold a 3500 lb weight in place as shown below. Determine the tension in each cable.

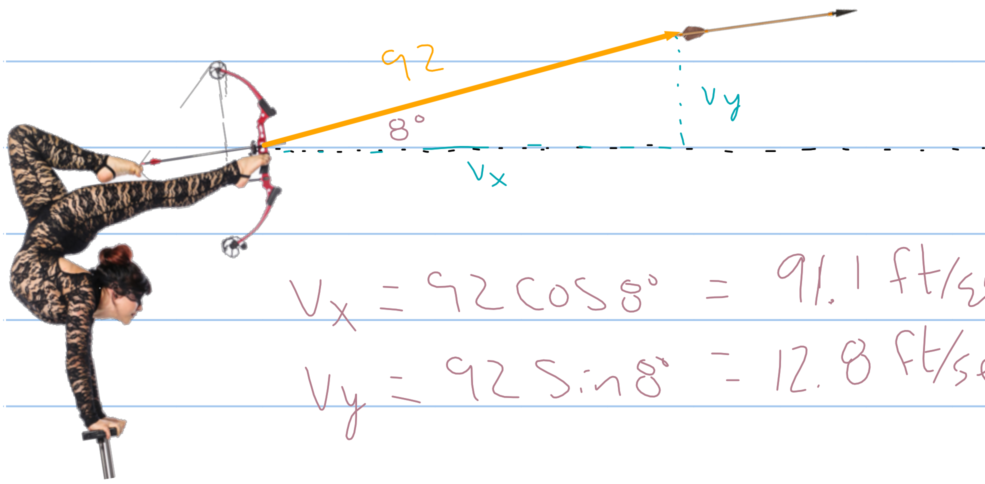




$$\frac{3500}{\sin 100°} = \frac{T_1}{\sin 50°} \qquad \frac{3500}{\sin 100°} = \frac{T_2}{\sin 30°}$$

$$T_1 = \frac{3500 \sin 50°}{\sin 100°} = 2722.5 \text{ lb} \qquad T_2 = \frac{3500 \sin 30°}{\sin 100°} \approx 1777.0 \text{ lb}$$

Example 7. An archer shoots an arrow at 8° from horizontal with a velocity of 92 feet per second. Find the vertical and horizontal components of the arrows velocity.



$$V_x = 92\cos 8^{\circ} = 91.1 \text{ ft/sec}$$

$$V_y = 92\sin 8^{\circ} = 12.8 \text{ ft/sec}$$