

$$f(x) = \cos(\sin(4x)) - 1.25$$

$$g(x) = -(9x^2 - 8)^{\frac{1}{3}}$$

- Let R be the region Bounded by $f(x)$ and $g(x)$ on the interval $[-0.947, 0.947]$
- The cross sections used to create a solid in R are right trapezoids. The cross sections are perpendicular to the x -axis with $\text{base}_1 = g(x) - f(x)$, $\text{base}_2 = .75 * \text{base}_1$, and $\text{height} = 0.5 * \text{base}_1$.
- The area of one cross section is: $A = \frac{1}{2} \cdot h(b_1 + b_2) = \frac{1}{4} b_1 \left(b_1 + \frac{3}{4} b_1 \right) = \frac{7}{16} (b_1)^2$
 where h = height of cross section
 b_1 = base_1 of cross section
 b_2 = base_2 of cross section

$$b_1 = -(9x^2 - 8)^{\frac{1}{3}} - \cos(\sin 4x) + 1.25$$

$$b_2 = .75 \left(-(9x^2 - 8)^{\frac{1}{3}} - \cos(\sin 4x) + 1.25 \right)$$

$$h = 0.5 \left(-(9x^2 - 8)^{\frac{1}{3}} - \cos(\sin(4x)) + 1.25 \right)$$

$$\text{Area of one cross section} = \frac{7}{16} \left(-(9x^2 - 8)^{\frac{1}{3}} - \cos(\sin 4x) + 1.25 \right)^2$$

- The volume of one cross section in the model is: $V = 0.0728A$
 Where A = Area of a cross section

$$0.0728 = \frac{0.947 + 0.947}{26} = \text{cross section thickness}$$

- The exact Volume of the solid is:

$$\frac{7}{16} \int_{-0.947}^{0.947} \left(-(9x^2 - 8)^{\frac{1}{3}} - \cos(\sin 4x) + 1.25 \right)^2 dx = 3.937919256 \text{ cubic units}$$

- Approximate volume using left Riemann Sum with 26 cross sections is:

x	Base 1 of trapazoidal cross section	Base 2 of trapazoidal cross section	Height of trapazoidal cross section	Area of trapazoidal cross section	Thickness of trapadoidal cross section	Volume of trapazoidal cross section
-0.947	0.011350633	0.008512974	0.005675316	5.63661E-05	0.0728	4.10345E-06
-0.8742	1.348977108	1.011732831	0.674488554	0.796135916	0.0728	0.057958695
-0.8014	1.556530931	1.167398198	0.778265465	1.059969986	0.0728	0.077165815
-0.7286	1.752284635	1.314213476	0.876142317	1.343344381	0.0728	0.097795471
-0.6558	1.974590662	1.480942996	0.987295331	1.705816123	0.0728	0.124183414
-0.583	2.204065253	1.65304894	1.102032626	2.125332842	0.0728	0.154724231
-0.5102	2.403646544	1.802734908	1.201823272	2.527663559	0.0728	0.184013907
-0.4374	2.541126737	1.905845053	1.270563368	2.825079728	0.0728	0.205665804
-0.3646	2.599269845	1.949452383	1.299634922	2.95583913	0.0728	0.215185089
-0.2918	2.577889574	1.933417181	1.288944787	2.907412662	0.0728	0.211659642
-0.219	2.494188597	1.870641448	1.247094298	2.721677331	0.0728	0.19813811
-0.1462	2.382378599	1.786783949	1.1911893	2.483130908	0.0728	0.18077193
-0.0734	2.287535877	1.715651908	1.143767938	2.289358919	0.0728	0.166665329
-0.0006	2.25000261	1.687501957	1.125001305	2.214848888	0.0728	0.161240999
0.0722	2.286365435	1.714774076	1.143182718	2.28701677	0.0728	0.166494821
0.145	2.380549543	1.785412158	1.190274772	2.479319556	0.0728	0.180494464
0.2178	2.492458436	1.869343827	1.246229218	2.717902712	0.0728	0.197863317
0.2906	2.576943245	1.932707434	1.288471622	2.905278463	0.0728	0.211504272
0.3634	2.599538797	1.949654098	1.299769399	2.956450857	0.0728	0.215229622
0.4362	2.542752059	1.907064044	1.27137603	2.828694765	0.0728	0.205928979
0.509	2.406486305	1.804864729	1.203243153	2.533639647	0.0728	0.184448966
0.5818	2.207701235	1.655775926	1.103850618	2.132350825	0.0728	0.15523514
0.6546	1.978408645	1.483806483	0.989204322	1.712419085	0.0728	0.124664109
0.7274	1.755737936	1.316803452	0.877868968	1.348644368	0.0728	0.09818131
0.8002	1.559615656	1.169711742	0.779807828	1.064175435	0.0728	0.077471972
0.873	1.353244524	1.014933393	0.676622262	0.801180949	0.0728	0.058325973
		Total volume approximation for Left Rieman Sum				3.91101548